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Heat-transfer aspects of Stirling power generation using incinerator waste energy

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Abstract

The integration of a free-piston Stirling engine with linear alternator and an incinerator is able to effectively recover the waste energy and generate electrical power. In this study, a cycle-averaged heat transfer model is employed to investigate the performance of a free-piston Stirling engine installed on an incinerator. With the input of source and sink temperatures and other realistic heat transfer coefficients, the efficiency and the optimal power output are estimated, and the effect induced by internal and external irreversibilities is also evaluated. The proposed approach and modeling results presented in this study provide valuable information for engineers and designers to recover energy from small-scale incinerators. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Free-piston Stirling engine; Linear alternator; Incinerator; Thermal efficiency

1. Introduction

The harmony between environmental protection and economic growth has become a worldwide concern; there is an urgent need to effectively reuse waste energy. In order to recover the waste energy of an incinerator, it has been a common practice to convert the water into superheated steam by absorbing the otherwise wasted energy. The obtained steam can then be used to power a steam turbine and generate electricity.

Owing to the hydrogen chloride and other corrosive substances produced during

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Nomenclature

$A_{\text{out,h}}$	surface area of engine's hot end (m^2)
F_{R}	radiation shape factor viewing from hot end surface to flame
$(HA)_{\text{C}}$	heat transfer coefficient from inner fluid to cold end surface (W/K)
$(HA)_{\text{H}}$	heat transfer coefficient from hot end surface to inner fluid (W/K)
$(HA)_{\text{O}}$	heat transfer coefficient from cold end surface to liquid coolant (W/K)
$(HA)_{\text{T}}$	heat transfer coefficient of heat-leak (W/K)
P	power (W)
$\dot{Q}_{\text{C}}$	heat rejection rate from engine (W)
$\dot{Q}_{\text{H}}$	heat transfer rate to engine (W)
$\dot{Q}_{\text{T}}$	bypass heat leak (W)
T_{C}	temperature at cold end side (K)
T_{H}	temperature at hot end side (K)
T_{L}	lowest temperature of engine's working fluid (K)
T_{O}	temperature of coolant (K)
T_{R}	flame temperature (K)
T_{U}	highest temperature of engine's working fluid (K)
α	absorptivity
β	$T_{\text{C}}/T_{\text{H}}$
ε	$(HA)_{\text{C}}/(HA)_{\text{H}}$
γ	$T_{\text{U}}/T_{\text{H}}$
η	thermal efficiency
λ	$T_{\text{L}}/T_{\text{U}}$
μ	$(HA)_{\text{T}}/(HA)_{\text{H}}$
σ	Stefan-Boltzmann constant, $5.6699 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4$

waste combustion, the temperature of superheated steam generated from incinerators is usually limited to below 400°C [1]. The low turbine inlet temperature results in a low generation efficiency, only 10~15% cited by Otoma et al. [2]. The efficiency of a steam turbine could be further decreased if the turbine capacity is reduced due to the size reduction in the incinerator. This is because the friction loss of the steam turbine is proportional to the surfaces of the turbine blades and stator, and the surface per kg steam flow rate increases rapidly as the turbine capacity decreases. In addition, the rotor tip clearance becomes significant compared to the turbine diameter, and the tip clearance losses become relatively large.

Therefore, the electricity generation by using a steam turbine generation unit (steam turbine GENSET) for a small-scale incinerator (say less than 500 tons of refuse per day) is not appropriate.

Stirling engines, on the other hand, can retain high efficiency over power ranges

from a few tens of watts to several kilowatts and are very suitable to use as the prime mover to recover waste energy and generate electricity from a small or medium scale waste incinerator.

Stirling engines have been found to run on conventional fuels such as gasoline or natural gas for distributed power generation applications [3,4]. Because of their external combustion characteristics, Stirling engines are uniquely suited to use the heat from a solar dish concentrating system [5] or burning biomass [6].

Among many Stirling machines, the free-piston Stirling engine matched with a linear alternator has the most mechanical simplicity and the least mechanical loss in converting waste heat into electricity. Heat transfer losses thus become the major part of the total energy-conversion (heat to electricity) loss.

In this study, a heat transfer model is used to simulate the heat transfer losses of a free-piston Stirling/linear alternator receiving energy from an incinerator's waste heat.

2. Description of free-piston Stirling/linear alternator

The crank-driven Stirling engine has a long history of development and presents some challenging design problems, including power modulation, leakage of working fluid, isolation of lubricants, etc.

These challenging problems can be resolved if the complex power-train mechanism of a crank Stirling engine is replaced by a simple moving body called a free-piston. Free-piston machines with an attached linear alternator can be hermetically sealed so as to contain the working gas (helium or hydrogen) for extended periods [7]. Fig. 1 shows a typical layout of a free piston Stirling engine, where the piston oscillation causes the compression–expansion and the displacer serves to move the working gas between hot and cold heat exchangers to accomplish the thermal cycle, the detailed sequences of starting and associated movement were presented in Walker et al. [8].

The regenerator, heater, and cooler are gap-type heat exchangers. The annular gas between the displacer and its cylinder provides the flow passage for these exchanges. Both piston and displacer float on gas bearings in their cylinders and are resonated by mechanical springs, which are arranged to eliminate side loads on the bearings. The combination of gas springs to allow wear-free close fits on the pistons and the use of mechanical springs to give both resonant frequency and axial positioning is uniquely advantageous in that it confers high mechanical efficiency and long life.

The linear alternator is used to generate AC power, and an electronic controller/power conditioner is employed to provide engine stability and AC-to-DC conversion.

3. Heat transfer model

Since the heat source of a Stirling engine is external to the working fluid, thermal energy must be transferred into and out of the engine via heat exchangers at the hot

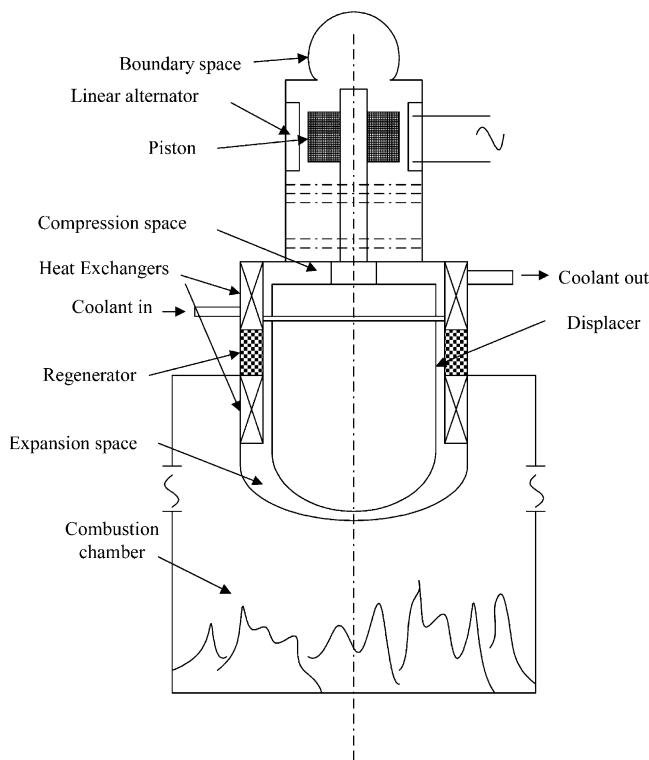


Fig. 1. Schematic of a free-piston Stirling engine on an incinerator.

and cold ends. There must be a temperature difference between the heat source and the working fluid when it receives thermal energy. In order to complete the thermal cycle, a temperature difference is required between the engine fluid and the heat sink when the engine rejects the thermal energy.

If a Stirling engine is installed in an incinerator, the hot end is situated inside the combustion chamber of the incinerator as shown in Fig. 1, the heat transfer model of this engine can be presented as that shown in Fig. 2.

The heat source T_R , shown in Fig. 2, represents the flame temperature inside an incinerator, and heat sink T_O stands for the temperature of the coolant used to cool down the engine's cold heat exchanger, see Fig. 1.

T_H is the temperature for the hot end side (the surface receiving flame heat), and T_C is the temperature for the other cold end side (contacting with coolant). T_U and T_L are, respectively, the highest and lowest temperatures of the engine's working fluid. The magnitude sequence of these temperatures is $T_R > T_H > T_U > T_L > T_C > T_O$.

The heat transfer from the heat source to the surface of the hot end is mostly contributed by radiation when T_R is relatively high, $\dot{Q}_H$ can be approximately written as

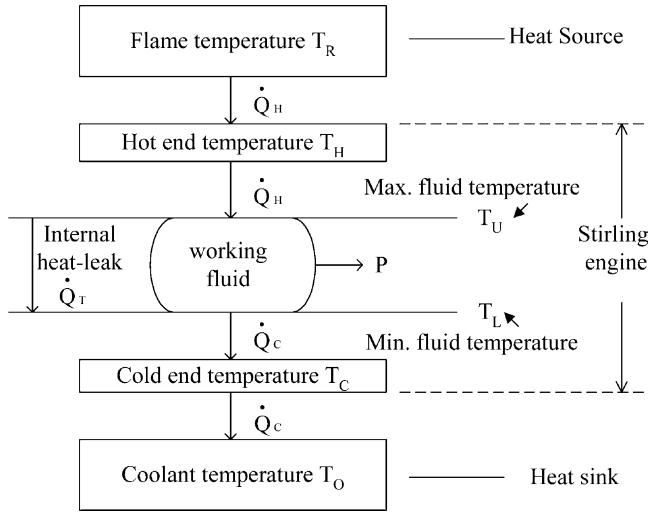


Fig. 2. Heat transfer model.

$$\dot{Q}_H = \alpha \sigma A_{\text{out},R} F_R (T_R^4 - T_H^4) \quad (1)$$

where σ =Stefan–Boltzmann constant, $5.6699 \times 10^{-8} \text{ W/m}^2 \times \text{K}^4$, α =absorptivity, $A_{\text{out},h}$ =surface area of the engine's hot end, F_R =radiation shape factor viewing from the hot end surface to the flame.

The heat transfer from the outer surface of the hot end to the inner working fluid is:

$$\dot{Q}_H = (HA)_H (T_H - T_U) \quad (2)$$

$(HA)_H$ is the reciprocal of the total thermal resistance from the outer solid surface to the inner fluid. The thermal resistance of a high conducting metal (stainless steel or Inconel alloy) is usually much less than the convection resistance of the working fluid (usually helium gas). $(HA)_H$ can be simplified to $(HA)_H \approx h_{\text{He}} A_{\text{in},h}$, where h_{He} is the convection coefficient of helium gas and $A_{\text{in},h}$ is the inner surface of the engine's hot space. To facilitate a higher heat transfer rate, $A_{\text{in},h}$ is usually fabricated into a fin-shape (microfin, etc.) such that $A_{\text{in},h} \gg A_{\text{out},h}$.

Similarly, the heat rejection from the working fluid to the outer surface of the cold end is:

$$\dot{Q}_C = (HA)_C (T_L - T_C) \quad (3)$$

where $(HA)_C \approx h_{\text{He}} A_{\text{in},C}$ and $A_{\text{in},C}$ is the inner surface of the engine's cold space.

The heat transfer from the cold end surface to the coolant is simply a convective heat transfer mode:

$$\dot{Q}_C = (HA)_O (T_L - T_O) \quad (4)$$

where $(HA)_O = h_{\text{coolant}} A_{\text{out,c}}$, h_{coolant} is the convective coefficient of the liquid coolant, and $A_{\text{out,c}}$ is the outer surface of the cold space.

The convective heat transfer coefficient of the liquid is almost one order of magnitude larger than that of the gas. But the surface cooled by the liquid is usually unfinned. In this analysis, we set $(HA)_O = (HA)_C$ for simplicity.

The heat leak through the engine is assumed to take place between the two internal extreme temperatures T_U and T_L at the rate of

$$\dot{Q}_T = (HA)_T(T_U - T_L) \quad (5)$$

In a typical Stirling engine, the heat leaks consist of the conduction through the structure connecting the hot and the cold sections (including regenerator, displacer etc.), through the working fluid itself and other useless heat such as the magnetic hysteresis loss, frictional loss etc.

The heat transfer rates presented in Eqs. (1)–(5) are the cycle-time-averaged formulations, the actual heat transfers may take place at different certain fractions of a complete thermal cycle.

4. Optimization of operating condition

With the definition of heat leak in Eq. (5), the cycle average power, P , is given by

$$P = (\dot{Q}_H - \dot{Q}_T) - (\dot{Q}_C - \dot{Q}_T) = \dot{Q}_H - \dot{Q}_C = (HA)_H(T_H - T_U) - (HA)_C(T_L - T_C) \quad (6)$$

The ideal Stirling engine comprises all reversible processes and its thermal efficiency is the same as the Carnot thermal efficiency [9]. In reality, there exists internal as well as external irreversibilities. The external irreversibilities (in our current heat transfer model) are considered via the finite temperature differences between the heat source T_R and the hot end T_H , and between the heat sink T_O and the cold end T_C . The heat leak shown in Eq. (5) can be considered as the representation of the overall internal irreversibility of this model.

Since $\dot{Q}_T$ stands for the overall internal irreversibility, the thermal efficiency η can be written as

$$\eta = \frac{P}{\dot{Q}_H - \dot{Q}_T} \leq 1 - \frac{T_L}{T_U} \quad (7)$$

In seeking the maximum possible power, it suffices to consider only the values when equality holds in Eq. (7). When $\dot{Q}_T = 0$ the cycle is internally reversible (or called the endoreversible thermal cycle). Eq. (7) also expresses the efficiency of a thermal cycle determined by the temperatures of the working fluid rather than the temperatures of the solid surfaces.

If we let $\lambda = \frac{T_L}{T_U}$, $\beta = \frac{T_C}{T_H}$, $\gamma = \frac{T_U}{T_H}$, $\varepsilon = \frac{(HA)_C}{(HA)_H}$, and $\mu = \frac{(HA)_T}{(HA)_H}$. From Eqs. (6) and (7), the engine power can be written as

$$P = (HA)_H T_H \frac{\varepsilon(\lambda - \beta)(1 - \lambda) - \mu(1 + \varepsilon\beta)(1 - \lambda^2)}{(\varepsilon + 1)\lambda - \mu(1 - \lambda)^2} \quad (8)$$

Eq. (8) gives the power output of the engine's cycle operating between the temperature extremes of the system fluid T_U and T_L , now represented as $\lambda = (T_L/T_U)$.

Within the Stirling engine, see the subregion bounded by extreme fluid temperatures T_U and T_L in Fig. 2, the output power P is only a function of the temperature ratio λ and the heat-leak ratio μ .

Differentiating Eq. (8) with respect to λ and setting it to zero yields the optimal λ value as

$$\lambda_M = \frac{\varepsilon\mu(1 - \beta) + \sqrt{(1 + \varepsilon)(\varepsilon + \mu + \mu\varepsilon)(\mu + \mu\varepsilon^2\beta^2 + \varepsilon\beta(1 + \varepsilon + 2\mu))}}{\varepsilon + \varepsilon^2 + \mu + 2\varepsilon\mu + \beta\varepsilon^2\mu} \quad (9)$$

With λ_M , the optimal power can be obtained from Eq. (8) for each fixed heat leak ratio.

Due to the polynomial nature of Eq. (1) and Eq. (9), iteration is needed in the solution procedure (see Fig. 3). Under the fixed source and sink temperatures and an estimated heat leak parameter, the optimal power, optimal efficiency, and associated temperatures can be obtained from the above mentioned equations.

5. Results and discussion

Fig. 4 shows the engine's power and efficiency vs. source temperature at different heat-leak ratios. As expected, both power and efficiency increase with T_R . The effect due to heat-leak loss is substantial. Under a 10% heat-leak ($\mu = 0.1$), the output power drops 30%, and the efficiency decreases from 33% to 23% when $T_R = 1400$ K.

In these calculations, sink temperature T_O is fixed at 300 K, a square meter of hot end surface with absorptivity times shape-factor equal to 0.85 ($\alpha F_R = 0.85$) is used as the calculation base.

As T_R increases, more heat can be absorbed by the hot end surface, however, the actual heat transfer rate (the bottle neck of $\dot{Q}_H$) is limited by the convective heat transfer coefficient of the working fluid (helium gas). The h_{HE} value is only about 20~200 W/m²K, the overall heat transfer coefficient is usually enhanced by increasing the finned surfaces on the gas side, which means $A_{in,h} > A_{out,h}$. In this study, $h_{HE} \cdot A_{in,h}$ is set to be $(HA)_H$; the values are ranged as 25~250 W/K, and the heat transfer coefficient of the cold end $h_{HE} A_{in,c}$ is set equal to $h_{HE} A_{in,h}$.

Fig. 5 depicts the variation of $\dot{Q}_H$, P and η when $(HA)_H$ is changed at fixed T_R (1400 K) and fixed T_O (300 K) values. As expected, $\dot{Q}_H$ increases with $(HA)_H$, and the increasing rate of $\dot{Q}_H$ declines at higher heat transfer coefficients.

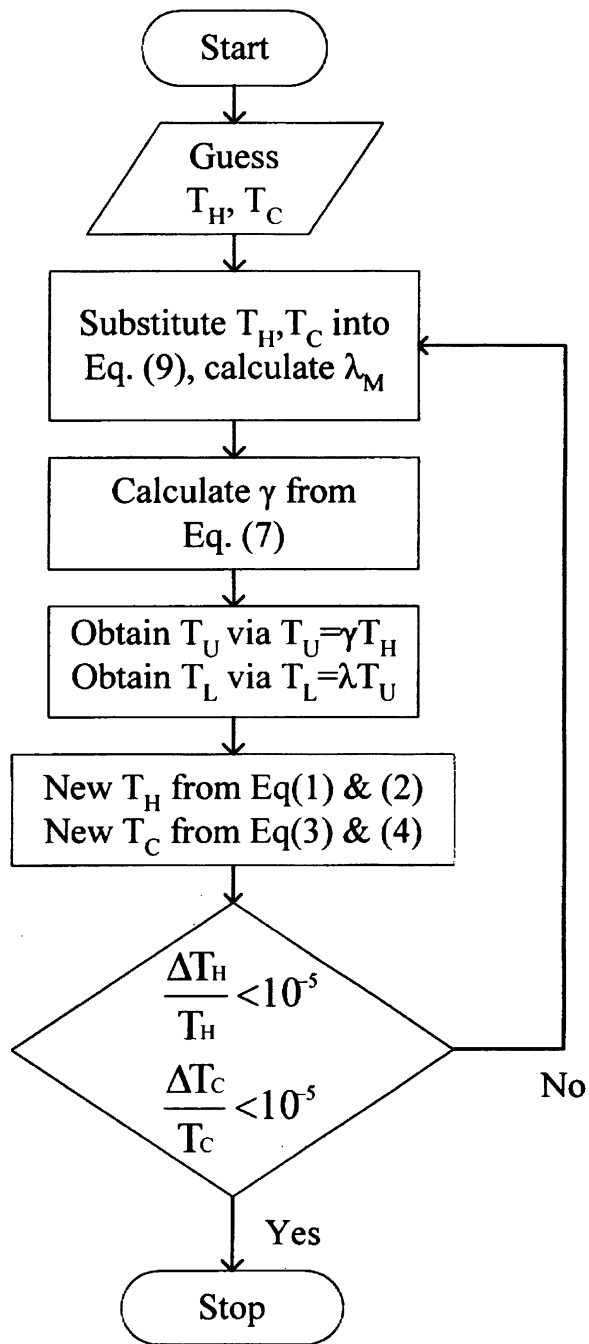


Fig. 3. Solution procedure.

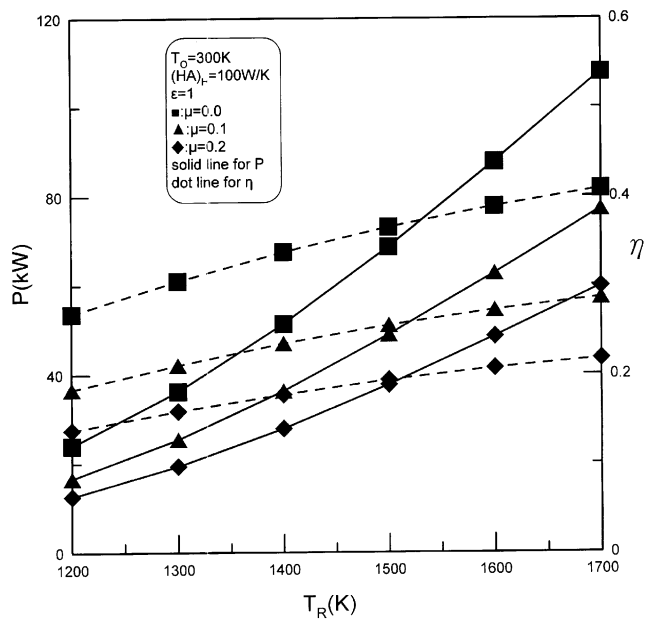
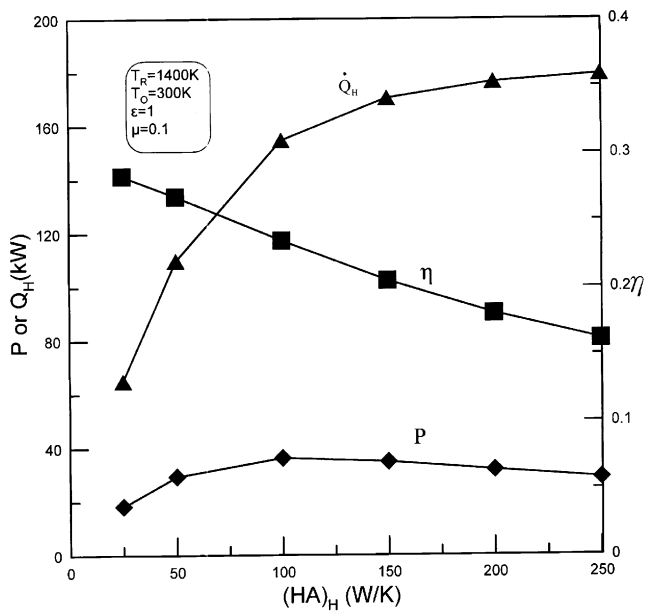


Fig. 4. Power and efficiency vs. source temperature at different heat leak.

Fig. 5. Heat source radiation rate, power and efficiency vs. $(HA)_H$.

Under a fixed T_R and T_O , the temperatures of the engine from the hot end surface to the cold end surface are decreasing when $(HA)_H$ is increasing. The differences between T_H and T_U as well as between T_L and T_C become smaller. A smaller temperature difference means a smaller driving potential that results in a lower efficiency, therefore η is gradually decreasing as shown in Fig. 5.

The net power output is merely the product of heat transfer rate times thermal efficiency. From the trend of $\dot{Q}_H$ and η , the resulting power value initially rises upward and turns slowly downward after reaching a peak.

The thermal efficiency of a Carnot engine (internally and externally reversible) is $\eta_{\text{Carnot}} = 1 - (T_O/T_R)$, which is about 78.57% when the source temperature $T_R = 1400$ K and the sink temperature $T_O = 300$ K. Curzon and Ahlborn [10] modified Carnot efficiency into $\eta_{CA} = 1 - \sqrt{T_O/T_R}$ for internally reversible engines, $\eta_{CA} = 53.7\%$ in the same temperature limits. From our calculations, see Fig. 5, the more realistic thermal efficiency is about 20~35% if $(HA)_H$ is 50~100 W/K, the associated output power is about 30~40 kW.

Compared to the solar Stirling engine from Stirling Energy Systems [5], which promoted a 25 kW genset which required a very expensive solar concentrator (100 m² parabolic dish with a two-axes tracking device), the generation unit proposed in this study requires less than one meter square hot end surface in order to produce 25 kW of electricity.

6. Conclusion

The cycle-averaged heat transfer model cannot predict the dynamic behavior of the actual Stirling engine, but can still provide some valuable information for designers, who may consider integrating a free-piston Stirling engine and an incinerator.

With less than 1 m² hot end surface embedded inside the combustion chamber of the incinerator, the absorbed energy is enough to move a 25 kW Stirling genset, therefore many similar units can be installed in parallel on the same incinerator. The modularity and standardization for a typical generation unit may substantially reduce the fixed cost and become economically attractive to the potential investors.

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